Mark schemes

Q1.

(a) (No external torque acts so) conservation of angular momentum applies $_1$✓

Can be expressed in symbols eg $I_T\ \omega_T = -(I_P\ \omega_P)$

Ang. momentum of turntable equal and opposite to ang. momentum of propeller so turntable rotates anticlockwise/in opposite direction to propeller $_2$✓

For $_2$✓ and $_3$✓ there must be correct statement and reason.

$I_T > I_P$ so turntable rotates more slowly than propeller $_3$✓

Do not allow $mass_T > mass_P$ instead of $I_T > I_P$

3

(b) **The mark scheme gives some guidance as to what statements are expected to be seen in a 1- or 2-mark (L1), 3- or 4-mark (L2) and 5- or 6-mark (L3) answer. Guidance provided in section 3.10 of the 'Mark Scheme Instructions' document should be used to assist marking this question.**

Mark	Criteria
6	All three areas covered with at least two aspects covered in some detail. 6 marks can be awarded even if there is an error and/or parts of one aspect missing.
5	A fair attempt to analyse all three areas. If there are several errors or missing parts then 5 marks should be awarded.
4	Two areas successfully discussed, or one discussed and two others covered partially. Whilst there will be gaps, there should only be an occasional error.
3	One area discussed and one discussed partially, or all three covered partially. There are likely to be several errors and omissions in the discussion.
2	Only one area discussed, or makes a partial attempt at two areas.
1	One of the three areas covered without significant error.
0	No relevant analysis.

Measuring angular speeds (ω_1 and ω_2) (Both required for full)
1A.Use of optical or magnetic sensor
2A. with datalogger/data-recorder/ more detail
1B. use of video /mobile phone/stroboscope
2B. with more detail
1C. stopwatch and fiducial mark
2C. and finding time for several revs

Other measurements (Both required for full)
3. Mass of dropped mass **AND** Distance of (centre of) mass to centre of turntable/axis of rotation
4A. using top pan balance/scales
4B. using vernier callipers or ruler

Calculation (Three points for full; one/two for partial)
5. idea that rev s^{-1} must be converted to rad s^{-1}
6A. M of I of mass = mr^2 (= I_{mass})
7A. $(I_{mass} + I)\ \omega_2 = I\ \omega_1$
6A & 7A. $I = \frac{mr^2\omega_2}{(\omega_1 - \omega_2)}$ OR substitute ω_1 ω_2 m and r into above equations to find I

A graphical method might be described, eg
6B. Plot $\frac{\omega_1}{\omega_2}$ against r^2 for various r

7B. $I = \frac{m}{\text{gradient}}$

6

[9]

Q2.

(a) Use of total area under graph = 2.52 rad ✓

Valid attempt at calculating the area in **Figure 2**✓

ω_{max} = 0.87(1) rad s^{-1} ✓

MP2 eg:
2.52 = [½ (1.5 × 0.35)] + [½ (ω_{max} + 0.35) × 1.2] + [½ × 3.5 × ω_{max}]
Must show calculation answer to 2 sf or more.

3

(b) Idea that $T = I\alpha$ must be used at times where there is a peak of the **Figure 3** graph. ✓

Calculates one value of $I\alpha$ at the time during any of the 3 accelerations where I is at a peak ✓

Calculates the maximum values of torque for all three acceleration and shows overall maximum value is at t = 2.1 s ✓

MP1: must show that they have considered using peak at 2.1 s and at least one other peak

1st acctn: T_{max} = 12200 × 0.35/1.5 = 2850 N m (at 0.65 s)
2nd acctn: T_{max} = 10400 × (0.87 - 0.35)/1.2 = 4510 Nm at 2.1 s
3rd acctn: T_{max} = 12800 × -0.87/3.5 = -3180 N m (at 5.1 s)
Allow ± 200 kg m^2 on I

Allow use of ω_{max} = 0.9 rad s^{-1} giving
2nd acctn T_{max} = 4760 N m
3rd acctn T_{max} = 3320 N m

Where comparisons are made condone answers that miss × 10^3 factor in I

For MP3, condone a calculation of max values of torque for both accelerations up to t = 2.7 s with a mention of the negative acceleration from t = 2.7s.

3

(c) ALTERNATIVE 1

at times near 2.7 s:

$T = I\alpha$; I is slightly smaller and α is constant so that torque is slightly smaller

OR

$P = T\omega$; T is slightly smaller but ω is increasing (as α positive). ✓

Both statements together with conclusion that Pmax must occur at a time other than 2.1 s ✓

ALTERNATIVE 2

Calculates power at t = 2.1 s ✓
Calculates power at a different time and concludes that P is greater at this time ✓

Answer does not have to be fully quantitative eg between 2.1 s and 2.7 s the torque decreases slightly because I decreases slightly and α remains constant.

But ω increases greatly to a max at 2.7 s, P = Tω so P greater at 2.7 s.

Ignore friction torque

Condone POT errors when a comparison is made.

2

[8]

Q3.

(a) torque ₁✓ = <u>moment of</u> inertia ₂✓ × <u>angular</u> acceleration ₃ ✓

1✓ 2✓ 3✓ 2 marks

any two of ₁✓ ₂✓ ₃✓ 1 mark

Do not accept 'inertia' for 'moment of inertia' or 'acceleration' or 'rotational acceleration' for 'angular acceleration'

If = or equals, or × or multiplied by, is missing or in wrong place: 0 marks

Condone 'proportional to' for '='.

2

(b) Refers to $I = \sum mr^2$ in symbols or in words ✓

(Plate) **A** because more of the mass is distributed at a greater distance from the axis of rotation (than for plate **B**)✓

Look for I depending on radius/distance of (constituent) masses squared.

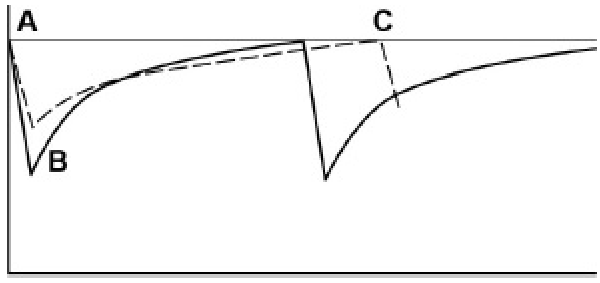
Do not accept '$I=mr^2$ ' alone

***A** alone is not enough. There must be a statement about the mass distribution or spread.*

2

(c) decrease in speed from ω_A not as much / ω_B greater ✓

curved line (from new **B**) with smaller initial gradient up to ω_A ✓



*Slope of new **AB** can be ignored unless it is shown with a much smaller gradient.*

Position of new C can vary, but shape of cycle must be similar to original.

*Do not mark beyond candidate's first **C***

2

(d) Either:

For same energy, in $E \frac{1}{2} I (\omega_A^2 \omega_B^2)$, (new) ω_B will be greater for greater I and same E ✓

Accept same idea expressed in other ways, eg initial (rotational kinetic) energy greater, so less fall in speed for same energy loss (during punching).

OR

For same motor torque, greater I means smaller angular acceleration, so smaller initial slope for **BC** ✓

*If **B** to **C** curve starts at same ω_B as Figure 2 with smaller initial gradient, and their **B** to **C** takes longer time to ω_A allow:*

*For same motor torque, greater I means lower angular acceleration, so longer time for **B** to **C***

1

[7]

Q4.

(a) $T = F \times r$ applies with some explanation of r in this context. ✓

force on pedal will vary/down force greater than up force/operator cannot keep force constant
OR
Component of force varies with position/rotation of crank
OR
Distance/radius of line of action of F from axle varies (as crank moves) ✓

Simply quoting $T = F \times r$ is not enough for MP1

2nd mark: accept: use of the term 'dead centre'
'Radius varies' is not enough

Accept idea that moment of force/torque varies with position of crank

2

(b) Relates angular impulse to change in angular momentum and to torque × time ✓

Accept answer using formulae:
Application of $T\Delta t = \Delta(I\omega)$ with understanding of symbols shown
Accept $Tt = \Delta I\omega$
Angular impulse does not have to be seen if $Tt = \Delta I\omega$ is correctly applied

If angular momentum/angular velocity reduced to zero in short time, high torque results (which will strain mechanism) ✓

$\Delta I\omega$ is fixed, t small, so T high (enough to strain mechanism)

2

(c) $\alpha = 13.8 \div 15.0 = 0.92$ (rad s^{-2}) ✓

Calculates I using $T = I\alpha$ giving 0.84 (kg m^2) ✓

OR

$\theta = \frac{1}{2}(w_1 + w_2)t = \frac{1}{2} \times 13.8 \times 15 = 103.5$ (rad)

Calculates I using $\frac{1}{2}I\omega^2 = T\theta$ giving 0.84 (kg m^2)

$I = \frac{0.77}{0.92} = 0.84$ kg m^2
ECF for incorrect α
$I = \frac{0.77 \times 103.5}{95.2} = 0.84$ kg m^2
ECF from incorrect θ (e.g. use of max speed instead of average speed)

2

(d) Equates 3.1 N m to 'sharpening' torque + frictional torque ✓

calculates sharpening torque and equates to $F \times r$ to give $F = 9.7$ N ✓

'sharpening' torque = 2.33 N m

Condone ECF for MP2 (but do not give MP1) if total torque is added to frictional torque, but neither MP1 nor MP2 given for ignoring friction torque or using only frictional torque. Expect to see F × 0.24 = 3.87 giving F = 16.1 N m.

No marks for F × 0.24 = 0.77 giving F = 3.2 N m

No marks for F × 0.24 = 3.1 giving F = 12.9 N m

2

(e) calculates any power by multiplying any **corresponding** $T \times \omega$ from any graph ✓

does this at $\frac{1}{2}\omega_0$ for 2 or 3 of the motors ✓

shows **F** is only motor to satisfy $\frac{2}{3}P_{max}$ criteria ✓

OR

calculates any power by multiplying any **corresponding** $T \times \omega$ from any graph ✓

calculates max power output of required motor as 1.5 × 52 = 78 W ✓

Shows only motor **F** satisfies this at $\frac{1}{2}\omega_0$ ✓

*For **E**: P_{max} = 120 × 0.43 = 52 W*

$\frac{2}{3}$ *× 52 = 35 W*

*For **F**: P_{max} = 150 × 0.52 = 78 W*

$\frac{2}{3}$ *× 78 = 52 W*

*For **G**: P_{max} = 90 × 0.64 = 58 W*

$\frac{2}{3}$ *× 58 = 39 W*

3

[11]

Q5.

(a) Attempt at calculating area above or below t axis or both ✓

(Ang displacement =) 2.80 + 2.10 − 3.15 = 1.75 rad

($\frac{1.75}{12.0}$ =) 0.15 (rad s^{-1}) ✓

Method must be valid

MP2: correct answer only

(calculator value = 0.145833)

MAX1 if counting square method used and answer rounds to 0.15 (rad s^{-1})

2

(b) $P = T\omega$ giving 546 (W) ✓

Allow ecf for 590 (W) from

using ω_1 = 1.5 rad s^{-1}

1

(c) Selects steepest part of graph and

determines gradient $\alpha = \frac{1.40 - -0.90}{5.0}$ = 0.46 (rad s^{-2}) ✓$_1$

$T = I\alpha$ = 9660 N m ✓$_2$

Adds friction torque to give 10 100 (N m) ✓$_3$

Accept any correct calculation of steepest graph slope: eg from 2 s to 5 s

$a = \frac{1.4}{3.0}$ *= 0.467 giving T = 9800 N m*

or 5 s to 7 s

$a = \frac{0.9}{2.0}$ *= 0.45 giving T = 9450 N m*

Allow ECF from MP2 to MP3

Treat 10 000 (Nm) as a 2 sf answer if consistent with their working.

3

(d) (net) $T \times t$ = 9660 × 5.0 = 4.8 × 10^4 (N m s) ✓

OR

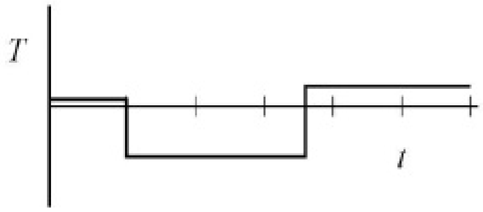
$\Delta(I\omega)$ = 2.1 × 10^4 (1.40 –(–0.90)) = 4.8 × 10^4 (N m s) ✓

*For first method allow ECF for torque ✓$_2$ from **(c)**, but not for ✓$_3$ value*

(calculator value = 48300)

1

(e)



✓

Tick (✓) against 3rd box

1

[8]

Q6.

(a) Attempt to use work done = force × distance with either incline work or resistance work or both ✓1

Work done by flywheel

= [(1.46 × 10^4 × 9.81 × sin 5°) + 1.18 × 10^3] × 500 ✓2

(= 6.83 × 10^6 J)

1/2 $I\omega^2$= 6.83 × 10^6 giving ω = 468 (rad s^{-1}) ✓3

MP1: award mark for valid attempt to calculate
mgh or F × s or both
mgh = 6.24 × 10^6 J
F × s = 5.9 × 10^5 J
MP2 for correct calculation of work done
MP3 for using their work done and 1/2 $I\omega^2$ to calculate ω
ECF for ✓3

3

(b) ✓1 for idea of use of flywheel as brake

✓2 for idea of storing and reusing this energy

E_p change of tram can be converted to E_k of flywheel so less energy transferred to brakes/brakes last longer/tram will not reach a high speed ✓1
OR
Energy otherwise dissipated/lost in brakes can be fed back to flywheel ✓1

Fly wheel is charged/stores energy and energy can be used for later acceleration/driving ✓2
OR

Fly wheel is charged/stores energy and at next stop less recharging energy will be needed. ✓2

Give two marks if both points covered in their answer to part 1

Treat as neutral answers in terms of providing a smoother ride or less wear on parts due to connecting and reconnecting flywheel.

If no other marks are given, allow 1 MAX for a correct reference to regenerative braking.

2

(c) **The mark scheme gives some guidance as to what statements are expected to be seen in a 1 or 2 mark (L1), 3 or 4 mark (L2) and 5 or 6 mark (L3) answer.**
Guidance provided in section 3.10 of the 'Mark Scheme Instructions' document should be used to assist in marking this question.

Mark	Criteria
6	The factors which affect E_k and all three areas of shape, material and design for high ω will be covered in some detail. 6 marks can be awarded even if there is an error and/or if parts of one aspect are missing.
5	The factors which affect E_k and all three areas will be covered, at least two in detail.
4	Two areas successfully discussed, or one discussed and two others covered partially. Whilst there will be gaps, there should only be an occasional error.
3	One area discussed and one discussed partially, or all three covered partially. There are likely to be several errors and omissions in the discussion.
2	Only one area discussed or makes a partial attempt at two areas.
1	None of the three areas covered without significant error.
0	No relevant analysis.

examples of the points made in the response

- E_k proportional to ω^2
- E_k proportional to I
- for same mass of tram I or ω increased but not mass of flywheel

Shape

- I depends on mass and distribution of mass around axis
- ($I = \sum mr^2$ so) arrange more m at outer edge of flywheel
- by using heavy rim and spokes/thin centre web
- increase radius

Material

- use higher density material at rim
- use material of higher tensile strength / breaking stress
- for higher speeds without bursting/to withstand rotational/centripetal stresses
- eg titanium, CFRP

Design for high ω - increase ω by:

- reduce friction at bearings
- use lubrication or roller bearings/air bearings/magnetic bearings
- smooth outer surfaces / encase in vacuum
- small increase in ω gives large increase in E_k (because ω^2)

Also allow

- sketches which convey correct info clearly
- use of 'depends on' for 'proportional to'
- need for perfect balance
- gyroscopic effects

6

[11]